Formalisation of nominal equational reasoning in PVS†

nominal anti-unification (the library nasa/pvslib/nominal)

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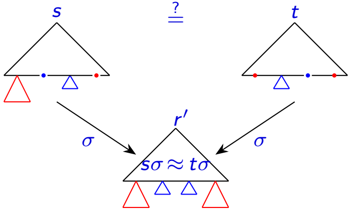
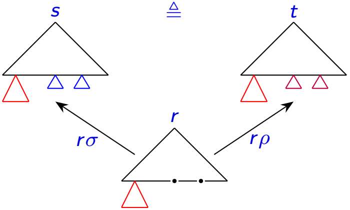
Outline

1. Motivation
2. Formal verification
3. Anti-unification modulo
4. A sound Algorithm for $(\mathfrak{a})(\mathsf{A})(\mathsf{C})(\mathfrak{a}\mathsf{A})(\mathfrak{a}\mathsf{C})$-theories
5. Future Work

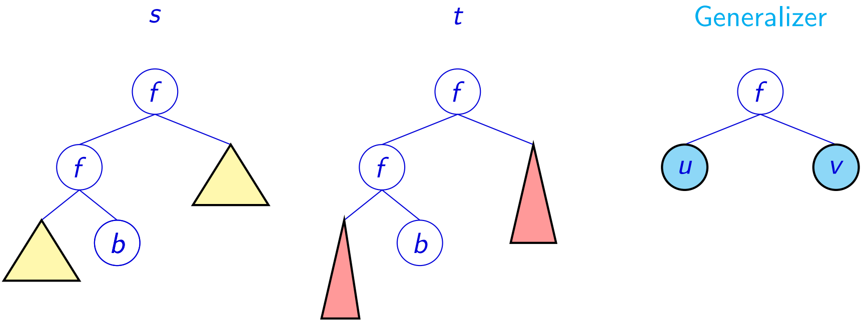
Unification Vs Anti-unification

Unification	Anti-unification
Goal: find a substitution that identifies two expressions.	Goal: find the commonalities between two expressions.

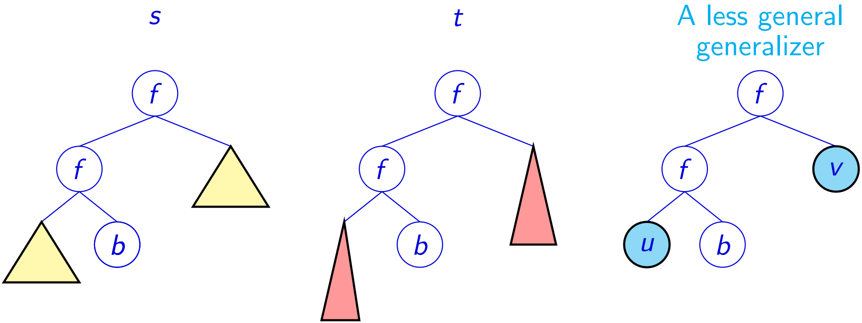




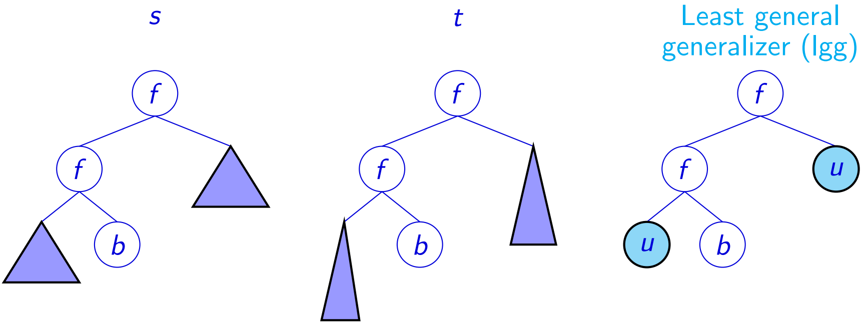
Anti-Unification



Anti-Unification



Anti-Unification



History

- Introduced by Gordon Plotkin [Plo70] and John Reynolds [Rey70]
- First-order: syntactic [Baa91]; C, A, and AC [AEEM14]; idempotent [CK20b], unital [CK20c], semirings [Cer20], absorption [ARCBK24]
- Higher-Order: patterns [BKLV17], top maximal and shallow generalizations variants [CK20a], equational patterns [CK19], modulo [CK20a]
- See david Cerna and Temur Kutsia survey [CK23].

Applications

Applications of anti-unification include:

- searching a large hypothesis space in inductive logic programming (ILP) for logic-based machine learning [CDEM22];
- preventing bugs and misconfigurations in software [MBK+20];
- detecting code clones [VY19];
- searching recursion schemes for efficient parallel compilation [BBH18].

Formal verification - Syntactical case

- terms $t ::= x \mid \langle\rangle \mid \langle t, t\rangle \mid f\ t$
- Labelled equations $E = \{s_i \underset{x_i}{\triangleq} t_i \mid i \leq n\}$

Configurations: $\langle E_U; E_S; \sigma\rangle$

$$\left\langle \underbrace{E_U}_{\text{Unsolved equations}} \quad \underbrace{E_S}_{\text{Solved equations}} \quad \underbrace{\sigma}_{\text{Substitution}} \right\rangle$$

Configuration constraints

- All labels in $E_U \cup E_S$ are different,
- no *redundant* equations appear in E_S, and
- no label in $E_U \cup E_S$ belongs to $dom(\sigma)$.

Inference Rules

$$\text{(Decompose Function)}\ \frac{\langle \{ f\ s \overset{\triangle}{\underset{x}{=}} f\ t \} \cup E, S, \sigma \rangle}{\langle \{ s \overset{\triangle}{\underset{y}{=}} t \} \cup E, S, \{ x \mapsto f\ y \} \circ \sigma \rangle}$$

$$\text{(Decompose Pair)}\ \frac{\langle \{ \langle s, u \rangle \overset{\triangle}{\underset{x}{=}} \langle t, v \rangle \} \cup E, S, \sigma \rangle}{\langle \{ s \overset{\triangle}{\underset{y}{=}} t, u \overset{\triangle}{\underset{z}{=}} v \} \cup E, S, \{ x \mapsto \langle y, z \rangle \} \circ \sigma \rangle}$$

$$\text{(Solve-Red)}\ \frac{\langle \{ s \overset{\triangle}{\underset{x}{=}} t \} \cup E, S, \sigma \rangle}{\langle E, S, \{ x \mapsto x' \} \circ \sigma \rangle}\ \text{if } s \overset{\triangle}{\underset{x'}{=}} t \in S$$

$$\text{(Solve-No-Red)}\ \frac{\langle \{ s \overset{\triangle}{\underset{x}{=}} t \} \cup E, S, \sigma \rangle}{\langle E, \{ s \overset{\triangle}{\underset{x}{=}} t \} \cup S, \sigma \rangle}\ \text{if there is no } s \overset{\triangle}{\underset{x'}{=}} t \in S$$

$$\text{(Syntactic)}\ \frac{\langle \{ s \overset{\triangle}{\underset{x}{=}} s \} \cup E, S, \sigma \rangle}{\langle E, S, \{ x \mapsto s \} \circ \sigma \rangle}\ \text{if neither decomposable nor solvable}$$

Inference Rules

Example

$$
\begin{array}{rc}
\text{(DecFun)} & \dfrac{\langle \{ f\langle f\langle c, b\rangle, c\rangle \triangleq_{x} f\langle f\langle d, b\rangle, d\rangle \}, \emptyset, id\rangle}{\langle \{ \langle f\langle c, b\rangle, c\rangle \triangleq_{y} \langle f\langle d, b\rangle, d\rangle \}, \emptyset, \{x \mapsto f\ y\}\rangle} \\
\text{(DecPair)} & \dfrac{}{\langle \{ f\langle c, b\rangle \triangleq_{z_1} f\langle d, b\rangle, c \triangleq_{z_2} d \}, \emptyset, \{x \mapsto f\ \langle z_1, z_2\rangle\}\rangle} \\
\text{(DecFun)} & \dfrac{}{\langle \{ \langle c, b\rangle \triangleq_{z_3} \langle d, b\rangle, c \triangleq_{z_2} d \}, \emptyset, \{x \mapsto f\ \langle f\ z_3, z_2\rangle\}\rangle} \\
\text{(DecPair)} & \dfrac{}{\langle \{ c \triangleq_{z} d, b \triangleq_{z_4} b, c \triangleq_{z_2} d \}, \emptyset, \{x \mapsto f\ \langle f\ \langle z, z_4\rangle, z_2\rangle\}\rangle} \\
\text{(SolveNRed)} & \dfrac{}{\langle \{ b \triangleq_{z_4} b, c \triangleq_{z_2} d \}, \{c \triangleq_{z} d\}, \{x \mapsto f\ \langle f\ \langle z, z_4\rangle, z_2\rangle\}\rangle} \\
\text{(Syntactic)} & \dfrac{}{\langle \{ c \triangleq_{z_2} d \}, \{c \triangleq_{z} d\}, \{x \mapsto f\ \langle f\ \langle z, b\rangle, z_2\rangle\}\rangle} \\
\text{(SolRed)} & \dfrac{}{\emptyset, \{c \triangleq_{z} d\}, \{x \mapsto f\ \langle f\ \langle z, b\rangle, z\rangle\}\rangle}
\end{array}
$$

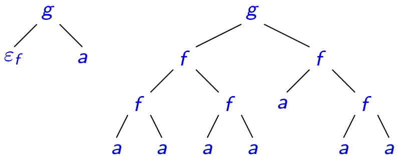
Anti-unification modulo

- Interest on the formalization of anti-unification for theories with Commutative, Associative and Absorption-symbols: C-, A-, and $\mathfrak{a}$-symbols.
- Related $\mathfrak{a}$-symbols are a pair of a function and a constant symbol holding the axioms $f(\varepsilon_f, x) = \varepsilon_f = f(x, \varepsilon_f)$.

Anti-unification in $(\mathfrak{a})(\mathrm{A})(\mathrm{C})(\mathfrak{a}\mathrm{A})(\mathfrak{a}\mathrm{C})$-theories

Example

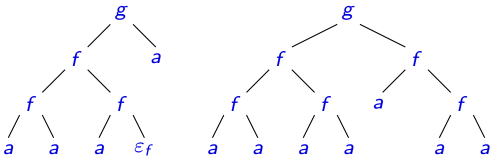
Consider the terms:



An $\mathfrak{a}$-generalization and $\mathfrak{a}$A-generalization will be illustrated.

Anti-unification in $(\mathfrak{a})(A)(C)(\mathfrak{a}A)(\mathfrak{a}C)$-theories

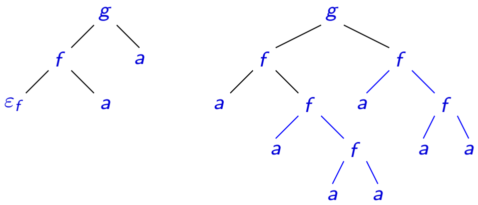
By *expanding* ε_f in $g(\varepsilon_f, a)$, one obtains:



Notice that $g(f(f(a,a), f(a,x)), y)$ is an $\mathfrak{a}$-generalization.

Anti-unification in ($\mathfrak{a}$)(A)(C)($\mathfrak{a}$A)($\mathfrak{a}$C)-theories

Considering the same terms modulo $\mathfrak{a}$A, and by *expanding* ε_f in $g(\varepsilon_f, a)$, one has:



$g(f(x, y), y)$ is an $\mathfrak{a}$A-generalization but not an $\mathfrak{a}$-generalization.

Anti-unification types for

Theory	Anti-unification type	References
Syntactic	1	[Plo70, Rey70]
A	ω	[AEEM14]
C	ω	[AEEM14]
Unital† $(\mathrm{U})^{1}$ / $(\mathrm{U})^{\geq 2}$	ω/nullary	[CK20c]
$\mathfrak{a}$	∞	[ARCBK24]

($\dagger$)Unital: $\{f(i_f, x) = f(x, i_f) = x\}$

Termination and Soundness

Termination

Syntactic anti-unification is terminating.

Soundness

For any valid configuration, syntactic anti-unification computes a least general generalizer.

Completeness

Any generalizer of a given input configuration is equal to or more general than the generalizer computed by syntactic anti-unification.

Conclusions and Future Work

Conclusions

- Although anti-unification has become of increasing interest, the verification of anti-unification algorithms has not been explored.
- The development of procedures to solve anti-unification modulo theories is crucial.
- Only recently, anti-unification modulo $\mathfrak{a}$-, C-, and ($\mathfrak{a}$C)-symbols have been addressed. Procedures combining such properties have been shown to be challenging from theoretical and practical perspectives.

Danke shön

Danke shön!

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